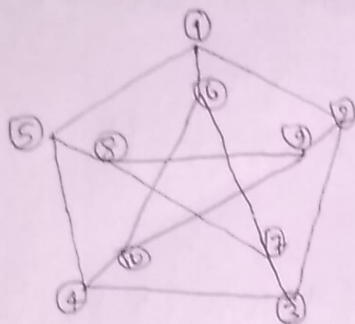


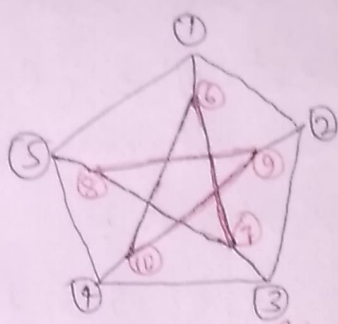
Graph Theory (SUBGRAPHS)

Let $G = (V, E)$. A graph $H = (V', E')$ is a subgraph of G if $V' \subseteq V$ and $E' \subseteq E$. The subgraph H is proper if $V' \subsetneq V$ or $E' \subsetneq E$.

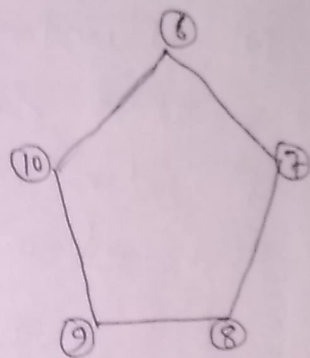
Example - we illustrate the notion of a sub-graph in following figure. Here we illustrate a sub-graph of the Petersen graph. The subgraph contains vertices 6, 7, 8, 9 and 10 and the edges containing them.



(a) Petersen graph



(b) Highlighted Subgraph



(c) Extracted Subgraph

The Petersen graph is shown (a) with a subgraph highlighted (b) and that the subgraph displayed on its own (c). A subgraph of a graph is another graph whose vertices and edges are sub-collections of those of the original graph.

Spanning Subgraph - Let $G = (V, E)$ be a graph and $H = (V', E')$ be a subgraph of G . The subgraph H is a spanning subgraph of G if $V' = V$.

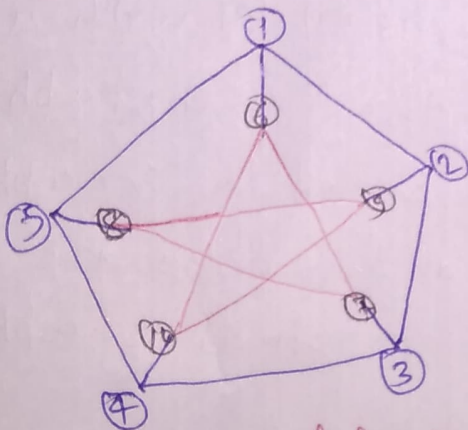
Edge Induced Subgraph - Let $G = (V, E)$ be a graph, if $E' \subseteq E$. The subgraph of G induced by E' is the graph $H = (V', E')$ where $v \in V'$ if and only if v appears in an edge in E' .

Vertex Induced Subgraph - Let $G = (V, E)$ be a graph. If $V' \subseteq V$. The subgraph of G induced by V' is the graph $H = (V', E')$ where $\{v_1, v_2\} \in E'$ if and only if v_1 and v_2 are both in V' .

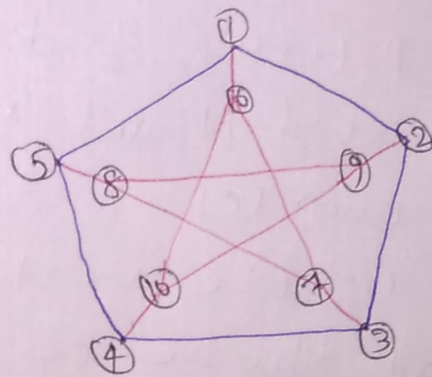
Note - For directed graphs, all subgraph definitions are modified in the obvious way. Edges become directed as one would expect.

Example - Using the Petersen graph we illustrate a subgraph induced by a vertex subset and a spanning subgraph. In figures below shown (a) illustrate the subgraph induced by the vertex subset $V' = \{6, 7, 8, 9, 10\}$ (b) illustrate spanning subgraph induced by the edge subset;

$$E' = \{ \{1, 6\}, \{2, 9\}, \{3, 7\}, \{4, 10\}, \{5, 8\}, \{6, 7\}, \{6, 10\}, \{7, 8\}, \{8, 9\}, \{9, 10\} \}$$



(a) Highlighted Subgraph



(b) Spanning Subgraph

The subgraph (a) is induced by the vertex subset $V' = \{6, 7, 8, 9, 10\}$. The subgraph shown in (b) is a spanning subgraph and is induced by the edge subset

$$E' = \{ \{1, 6\}, \{2, 9\}, \{3, 7\}, \{4, 10\}, \{5, 8\}, \{6, 7\}, \{6, 10\}, \{7, 8\}, \{8, 9\}, \{9, 10\} \}$$